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A prediction for the 25th solar cycle maximum amplitude

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The minimum - maximum method, belonging to the precursor class of the solar activity forecasting methods, is based on a linear relationship between relative sunspot number in the minimum and maximum epochs of solar cycles. In the present analysis we apply a modified version of this method using data not only from the minimum year, but also from a couple of years before and after the minimum. The revised 13-month smoothed monthly total sunspot number data set from SILSO/SIDC is used. Using data for solar cycle nos. 1-24 the largest correlation coefficient (CC) is obtained when correlating activity level 3 years before solar cycle minimum with the subsequent maximum ($CC = 0.82$), independent of inclusion or exclusion of the solar cycle no. 19. For the next solar maximum of the cycle no. 25 we predict: $R_{\max} = 121 \pm 33$. Our results indicate that the next solar maximum (of the cycle no. 25) will be of the similar amplitude as the previous one, or even something lower. This is in accordance with the general middle-term lowering of the solar activity after the secular maximum in the 20th century and consistent with the Gleissberg period of the solar activity. The reliability of the "3 years before the minimum" predictor is experimentally justified by the largest correlation coefficient and verified with the Student t-test. It is satisfactorily explained with the two empirical well-known findings: the extended solar cycle and the Waldmeier effect. Finally, we successfully reproduced the maxima of the last four solar cycles, nos. 21-25, using the 3 years before the minimum method.

KEYWORDS:
Sun: activity, sunspots

1 | INTRODUCTION

"Prediction is difficult, especially of the future." This quote is attributed to Niels Bohr (Peitgen, Jürgens, & Saupe, 2004). In this work we make a prediction of the amplitude of the next solar cycle maximum, knowing that the minimum between solar cycles no. 24 and no. 25 was in December 2019.

The solar magnetic activity cycle belongs to the few problems in contemporary solar physics which are not resolved to the satisfactory level. Besides their important place within solar physics (Brun & Browning, 2017; Charbonneau, 2013, 2014, 2020; Harvey, 1992; Hoyng, 1992; Miesch & Teweldebirhan, 2016; Ossendrijver, 2003; Rüdiger & Hollerbach, 2004; Thomas & Weiss, 2008; P. R. Wilson, 1994), they also have a practical role as drivers of the space weather (Koskinen et al., 2017) and possible causes of the climatic change (Gray et al., 2010; Rapp, 2008).

Moreover, solar cycle forecasting serves as a very good tool to estimate the upcoming geomagnetic activity, which is very important for planning the satellite missions within the Earth's magnetosphere, orbital correction of the satellites already in the orbits around the Earth, and protection of on board instruments that monitor the ionospheric-magnetospheric state. Verbanac, Mandea, Vršnak, & Sentic (2011) performed a thorough analysis of the relationship between various solar

and geomagnetic activity indices on 1-year data resolution. Based on their work, which demonstrated quite strong relationship between all considered geomagnetic and solar activity parameters, the 1-year prediction of the level of geomagnetic disturbances is possible. Namely, they found the average 1-year time delay of the geomagnetic activity (quantified by geomagnetic Ap index) behind all solar indices including sunspot numbers used in the present study. Further, the delay of Ap index for two years with respect to 10.7 cm flux is presented in Verbanac, Vršnak, Temmer, Mandea, & Korte (2010). Based on that it is clear that the prediction of the magnitude of the solar cycle is very useful in estimating the geomagnetic activity at least one year in advance.

There is also an additional, "purely astronomical" motivation for the precise monitoring of the solar activity and for developing reliable solar cycle prediction tools. It is widely known that the sky brightness is well correlated with the solar activity, in particular with the 10.7 cm flux. Solar EUV radiation, variable during the solar activity cycle, influences the airglow in the upper Earth's atmosphere, increasing the sky brightness during the maximum of activity (Walker, 1988). The effect should be taken into account for efficient planning of astronomical observations with large facilities, such as Rubin/LSST[1] (Ivezić et al., 2019) and ESO[2] telescopes (Leinert, Vaisanen, Mattila, & Lehtinen, 1995; Patat, 2003). Moreover, solar cycle prediction is also important for optimal observational plan of those solar phenomena which are strongly correlated with the solar activity, such as active regions, sunspots, and flares. This is again very important for large facilities, such as ALMA[3], where the observing time is sparse and spread over almost all types of astronomical objects, with solar observations having only a small fraction of the whole observing time (Brajša et al., 2018).

The most common index of solar activity is the sunspot number. After several hints and alerts that the sunspot number series in use needs to be revised for several inconsistencies, a serious program for recalibration of the sunspot number was started (Clette, Svalgaard, Vaquero, & Cliver, 2014; Cliver, Clette, & Svalgaard, 2013; Cliver, Clette, Svalgaard, & Vaquero, 2015) and successfully finished in the mid of 2015 (Clette & Lefèvre, 2016). There are several reasons why the "old" sunspot number should be corrected, the most important being (i) mutually inconsistent sunspot number series and group sunspot number series, (ii) more and more evidence that the sunspot number series is not homogenous showing important discontinuities obviously not representing real changes of the solar activity, and (iii) a curious secular trend in solar activity inferred from variations

of the sunspot number (Clette et al., 2014; Cliver et al., 2013). The recalibration process had to be done very carefully, since the sunspot number is widely used in studies of the solar dynamo, terrestrial climate change and space climate change (Cliver et al., 2015). Finally, the "new", improved sunspot number[4] was officially introduced on July 1st, 2015.

There are two general types of methods for the solar cycle predictions in use: the empirical methods and the methods relying on MHD dynamo models (Brajša et al., 2009; Hathaway, 2009, 2015; Hathaway, Wilson, & Reichmann, 1999; Pesnell, 2012; Petrovay, 2010, 2020). The empirical methods are further divided into the extrapolation and the precursor methods.

In present work we use the empirical approach, a modified minimum - maximum method, which belongs to the precursor class of methods. It was introduced by R. M. Wilson (1990b) and the input data include the sunspot number and various geomagnetic indices. Although the method is relatively simple, up to now it has not been used as much as it would be expected (Brajša et al., 2015; Brajša et al., 2009; Pishkalo, 2014; Ramesh & Lakshmi, 2012). Basically the method uses knowledge of some solar parameters or proxies in and around solar minimum to predict the level of activity in the next solar maximum. The solar parameter used in present work is the sunspot number. The modification used here consists of using the input data shifted in time from the minimum epoch.

Present work has three main aims. (i) To check whether the assumption that 3 years before the activity minimum is the best epoch to predict the next solar maximum is true. This assumption was made independently by Svalgaard, Cliver, & Kamide (2005) and by Cameron & Schüssler (2007) using different methods for solar cycle forecasting than in present work. Their methods and arguments will be discussed in more detail later in this paper. The second aim of this analysis is (ii) To make a prediction of the amplitude of the next solar cycle maximum, using the modified minimum - maximum method, taking into account the previous assumption, and knowing that the minimum between solar cycles no. 24 and no. 25 was in December 2019. Finally, (iii) we check the reliability of the method by reproducing the maxima of the last four solar cycles, nos. 21-25, using the proposed modified minimum - maximum method.

2 | DATA SET AND REDUCTION METHOD

We use the 13-month smoothed monthly total sunspot number data set for the period 1749 to March 2020 (which became

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[1] https://www.lsst.org/
[2] https://www.eso.org/public/teles-instr/
[3] http://www.almaobservatory.org

[4] http://sidc.oma.be/silso/newdataset

TABLE 1 The solar cycle number, the epochs of solar minima and maxima (T_{min} and T_{max}, respectively) and the extreme values of the monthly smoothed sunspot numbers in corresponding months (R_{min} and R_{max}, respectively).

Cycle No.	T_{min} (year)	(month)	T_{max} (year)	(month)	R_{min-3}	R_{min}	R_{max}
1	1755	3	1761	6	75.5	14.0	144.1
2	1766	6	1769	9	76.3	18.6	193.0
3	1775	6	1778	5	113.0	12.0	264.3
4	1784	9	1788	2	101.0	15.9	235.3
5	1798	4	1805	2	46.6	5.3	82.0
6	1810	7	1816	5	15.9	0.0	81.2
7	1823	5	1829	11	30.2	0.2	119.2
8	1833	11	1837	3	106.6	12.2	244.9
9	1843	7	1848	2	103.8	17.6	219.9
10	1855	12	1860	2	84.5	6.0	186.2
11	1867	3	1870	8	88.5	9.9	234.0
12	1878	12	1883	12	20.9	3.7	124.4
13	1890	3	1894	1	21.0	8.3	146.5
14	1902	1	1906	2	34.0	4.5	107.1
15	1913	7	1917	8	29.4	2.5	175.7
16	1923	8	1928	4	58.2	9.4	130.2
17	1933	9	1937	4	51.2	5.8	198.6
18	1944	2	1947	5	91.2	12.9	218.7
19	1954	4	1958	3	100.2	5.1	285.0
20	1964	10	1968	11	73.0	14.3	156.6
21	1976	3	1979	12	62.8	17.8	232.9
22	1986	9	1989	11	91.7	13.5	212.5
23	1996	8	2001	11	73.6	11.2	180.3
24	2008	12	2014	4	36.0	2.2	116.4
25	2019	12	–	–	28.5	1.8	–

available in October 2020), from the Sunspot Index and Long-term Solar Observations (SILSO[5]) Data Center of the ROB, Brussels (SILSO World Data Center, 1749-2020). The data from the solar cycle no. 1 up to minimum between solar cycles no. 24 and no. 25 are presented in Table 1 .

A modified version of the minimum - maximum method is used considering not only the minimum activity years, but also years before and after the epoch of the solar activity minimum. We use the input data for solar cycles nos. 1–24 from Table 1 and data of the epochs of minima from SILSO. The procedure is repeated excluding the data for solar cycle no. 19. The solar cycle no. 19 is a rather unusual cycle in which the highest measured activity maximum was preceded by a relatively low minimum (Table 1). There are many indications that this solar cycle was not a typical one, even maybe a real outlier (Temmer et al., 2006; R. M. Wilson, 1990a).

The correlation coefficient, CC, is investigated as a function of the time offset in years (Figure 1). A maximum in the

CC 3 years before the minimum is clearly seen in Figure 1 . After the minimum the CC sharply rises as the cycle moves toward the maximum. The two datasets converge and therefore the correlation approaches 1, $CC = 1$. It is interesting that the CC has the lowest value in the epoch of the solar minimum. A general behavior of the CC as a function of the offset without cycle no. 19 is similar to the case with all solar cycles, with the exception that the minimum in the CC value occurs one year after the activity minimum.

In Figure 2 the linear least-square fit for R_{max} vs. R_{min} is presented and in Figure 3 the similar fit for R_{max} vs. R_{min-3}, together with the 1σ uncertainty boundaries. R_{min-3} represents the smoothed monthly sunspot number 3 years before the activity minimum. Both procedures were performed with and without the unusual solar cycle no. 19. The exclusion of the cycle no. 19 gives slightly different fits (Figures 2 and 3).
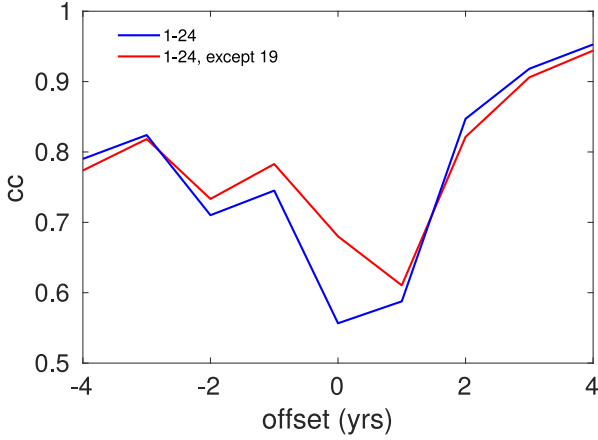
FIGURE 1 The correlation coefficient *CC* of the minimum − maximum relationship for different values of the time offset in years. Solar cycles nos. 1-24 were analyzed using monthly smoothed sunspot numbers. Results obtained with and without solar cycle no. 19 are presented with different colors, as indicated in the legend.
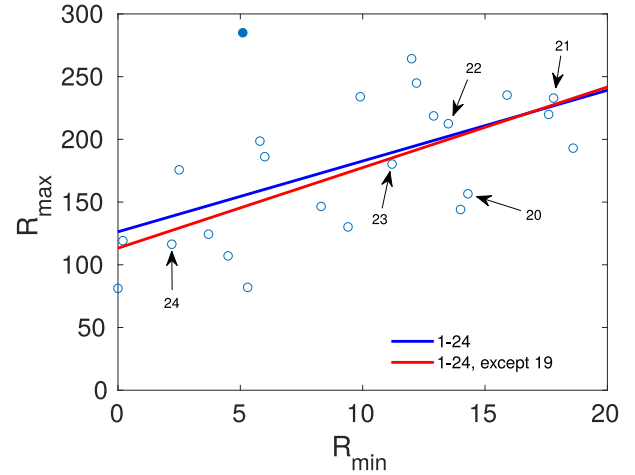

FIGURE 2 The peak smoothed monthly sunspot number in solar cycle maxima as a function of the same quantity in the preceding solar minimum, for solar cycles nos. 1-24. Least-square fits obtained with and without solar cycle no. 19 are presented with different colors, as indicated in the legend. The value for the solar cycle no. 19 is represented with the filled circle, while all other data points are represented with open circles.
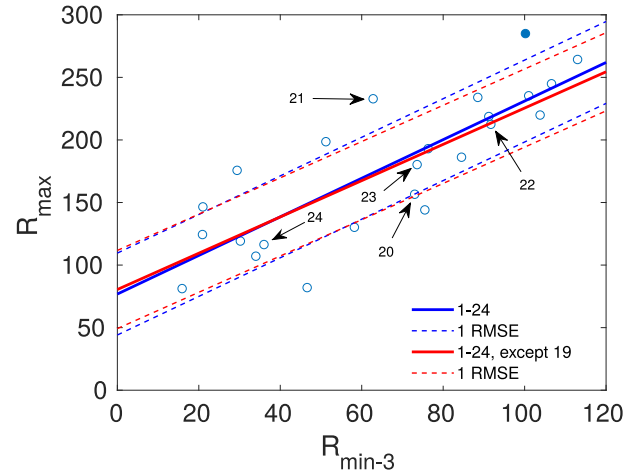
3 | RESULTS

We present the least-square fit parameters of the general form:

$$R_{max} = a \cdot R_{min}(t) + b \qquad (1)$$

where R_{max} is the amplitude of the cycle maximum and R_{min} is the activity value in the minimum or before. Taking into account input data for solar cycles $1 - 24$ we get:

$$R_{max} = (5.6 \pm 1.8) \cdot R_{min} + (126 \pm 20), \quad CC = 0.56 \quad (2)$$

$$R_{max} = (1.5 \pm 0.2) \cdot R_{min-3} + (77 \pm 17), \quad CC = 0.82 \quad (3)$$

We now repeat the formulae obtained without the solar cycle no. 19:

$$R_{max} = (6.4 \pm 1.5) \cdot R_{min} + (113 \pm 17), \quad CC = 0.68 \quad (4)$$

$$R_{max} = (1.4 \pm 0.2) \cdot R_{min-3} + (80 \pm 16), \quad CC = 0.82 \quad (5)$$

Taking into account the date of the current minimum (December 2019) we can use the observed monthly smoothed value $R_{min} = 1.8$ and the value three years before (December 2016) $R_{min-3} = 28.5$ to calculate the maximum of the current 25th solar cycle. For the four cases described earlier, the following predictions for the maximal amplitude of the solar cycle no. 25 are calculated using Equations (2) – (5), respectively:

$$R_{max} = 137 \pm 48 \quad (R_{min}, R_{max}, \text{nos.} \, 1 - 24) \qquad (6)$$

$$R_{max} = 121 \pm 33 \quad (R_{min-3}, R_{max}, \text{nos.} \, 1 - 24) \qquad (7)$$

$$R_{max} = 125 \pm 40 \quad (R_{min}, R_{max}, \text{excl. no.} \, 19) \qquad (8)$$

$$R_{max} = 122 \pm 31 \quad (R_{min-3}, R_{max}, \text{excl. no.} \, 19) \qquad (9)$$

FIGURE 3 Similar to Figure 2 , but for the values 3 years before the activity minima. Dashed lines represent uncertainty of the fits.

The given errors represent the RMSE. We see that excluding the solar cycle no. 19 narrows the prediction for the two subcases, minimum vs. maximum and minimum - 3 years vs. maximum.

To further investigate the predictive reliability of both approaches, we applied them to predict the maximum amplitude of the last four completed solar cycles nos. 21–24 and compare the result with the actual observed amplitude. As before, we considered cases with and without the solar cycle 19. For each of the four cycles tested, the linear fit coefficients were calculated by using data only from the cycles older than the one being predicted. Although this means that the number of data points available for fitting is decreasing for each past cycle, doing it this way better simulates actual past predictions when knowledge of future cycles was not available. This method also gives the stability of the fit coefficients during the last several cycles.

The results of the maximum amplitude prediction for the last four solar cycles, with addition of the next 25th cycle, and comparison with the actual measured amplitude are given in Table 2 and graphically presented in Figure A1 which is given in the Appendix. Upper two table sections show results from the minimum–maximum method, while lower two sections list values obtained using the 3 years before minimum method. It can be seen that the slope coefficient a remains pretty stable with its error generally slightly decreasing when more cycles are used. The same can be concluded for the intercept coefficient b, when the R_{min} vs R_{max} method is used, except in the last case when all 24 cycles are used and the value of b slightly decreases but is still fairly within 1σ. This is due to the very low minimum of the 24th cycle. When 3 years before the minimum method is used, b value again shows statistically insignificant variation.

On the other hand, the CC slowly increases in value when more and more cycles are used, for the minimum - maximum method. The only exception is cycle 21 (when included in the analysis) which worsens the correlation slightly for the 3 years before minimum method. As can be seen in Figure 3, cycle 21 falls far from the best fit line, more the 1σ away, thus decreasing the overall correlation. This is also the reason behind the 2σ difference in the predicted and observed values of the cycle 21. Furthermore, it is worth mentioning that the removal of the cycle 19 improves the RMSE of the minimum–maximum method but has no significant effect on the 3 years before the minimum method.

In summary, 3 years before the minimum method is preferred over the minimum–maximum method since it has better CC and lower statistical errors. This is also visible when comparing Figures 2 and 3. Data points are grouped more closer to the regression line in case of the 3 years before the minimum method. However, for a particular cycle it may happen (e.g. cycle 21) that a point is further away from the regression line for R_{min-3} than for R_{min}. In that case, R_{min} would give a better prediction, but it is a random event which depends on the particular cycle.

4 | DISCUSSION

First, we can confirm the assumption that 3 years before solar activity minimum is the best time when reliable prediction for the next maximum can be made. This conclusion is supported by the highest correlation coefficient at that epoch (Figure 1). So, the assumption of the importance of the time 3 years before minimum, made by Svalgaard et al. (2005) and Cameron & Schüssler (2007), is here independently confirmed and reaffirmed. Moreover, the curves for the two cases, with and without solar cycle no. 19, have almost the same value at the epoch 3 years before solar minimum (Figure 1). An important implication of this fact is that excluding the solar cycle no. 19 does not have a significant influence on the predictive skill of the method, if the modified procedure R_{max} vs. R_{min-3} is considered. Finally, we can also easily understand the minimal value of the correlation coefficient for the R_{max} vs. R_{min} case when solar cycle no. 19 is included (Figure 1). The explanation is based on the fact that the highest solar activity maximum observed in solar cycle no. 19 was preceded by a relatively low minimum (Table 1). This blurs the correlation, but the influence is completely removed when data 3 years before the minimum are considered.

A question can be raised if the difference between the minimum vs 3 years before the minimum CCs is statistically significant. To investigate this, we performed an unequal variances t–test (Ivezić, Connelly, VanderPlas, & Gray, 2014; Press, Teukolsky, Vetterling, & Flannery, 2002; Student, 1908; Welch, 1947) on two samples corresponding to two prediction methods. T-test is used to determine if two populations have equal means, i.e. are the means of two data sets significantly different from each other. This is numerically characterized by a t-value, a distance between the two means in terms of standard deviations, and a p-value, a probability of obtaining the observed, or more extreme, value when the null hypothesis is true. The null hypothesis, in our case, is that both population means are equal. For the R_{min} and R_{min-3} data sets, using all 24 cycles, the null hypothesis is strongly rejected by the t–test with a p-value well below the usual 0.05 threshold (1.5×10^{-11}). We also compared R_{min-3} and R_{min-t}, where t is time in years from the minimum epoch, for t values of 1 to 4 years. As expected, the t-test gave the largest p-values for R_{min-3} neighboring data sets (t equal to 2 and 4 years, corresponding p-values are 0.0026 and 0.0019, respectively), and increasingly smaller values for more distant data sets. From the analysis above, we can conclude that R_{min-3} and R_{min} data sets used for the prediction of the amplitude of the next maximum are statistically different and do not represent two samples of the same population.

TABLE 2 Comparison of the predicted and observed solar cycle maxima amplitudes for cycles 21-25. The column named "cycles used" denotes how many previous solar cycles were employed to calculate the fit coefficients (a, b) and their errors (σ_a, σ_b), as well as correlation coefficients (CC) and predicted amplitudes (R_{max}) of the next cycles based on the observed R_{min}/R_{min-3} of the same cycles.

Method	Cycles used	a	σ_a	b	σ_b	CC	Next cycle	$\dfrac{R_{min}}{R_{min-3}}$	Predicted R_{max}	RMSE	Observed R_{max}
	1-24	5.6	1.8	126.3	19.9	0.56	25	1.8	136.5	48.0	-
R_{min} vs R_{max}	1-23	5.4	1.9	129.6	21.5	0.53	24	2.2	141.4	48.8	116.4
with	1-22	5.4	2.0	129.8	22.0	0.53	23	11.2	190.6	49.9	180.3
cycle 19	1-21	5.4	2.1	129.9	22.6	0.52	22	13.5	202.4	51.1	212.5
	1-20	5.3	2.3	130.5	23.6	0.49	21	17.8	224.1	52.4	232.9
	1-24	6.4	1.5	113.3	17.0	0.68	25	1.8	124.8	39.8	-
R_{min} vs R_{max}	1-23	6.3	1.6	115.0	18.6	0.66	24	2.2	128.8	40.7	116.4
without	1-22	6.3	1.7	115.1	19.1	0.66	23	11.2	185.8	41.7	180.3
cycle 19	1-21	6.2	1.8	115.2	19.6	0.65	22	13.5	199.4	42.7	212.5
	1-20	6.1	1.9	115.8	20.5	0.62	21	17.8	225.0	43.8	232.9
	1-24	1.5	0.2	76.8	16.8	0.82	25	28.5	120.8	32.7	–
R_{min-3} vs R_{max}	1-23	1.5	0.2	79.1	17.8	0.82	24	36.0	133.8	33.3	116.4
with	1-22	1.5	0.2	79.4	18.2	0.82	23	73.6	191.4	34.0	180.3
cylce 19	1-21	1.5	0.3	79.2	18.7	0.81	22	91.7	219.5	34.8	212.5
	1-20	1.5	0.2	75.7	18.0	0.84	21	62.8	172.3	33.0	232.9
	1-24	1.4	0.2	80.5	16.2	0.82	25	28.5	121.8	31.2	–
R_{min-3} vs R_{max}	1-23	1.4	0.2	82.8	17.1	0.81	24	36.0	134.1	31.7	116.4
without	1-22	1.4	0.2	83.0	17.6	0.81	23	73.6	188.1	32.5	180.3
cycle 19	1-21	1.4	0.3	82.9	18.2	0.81	22	91.7	214.1	33.3	212.5
	1-20	1.4	0.2	79.5	17.1	0.83	21	62.8	169.6	31.0	232.9

Considering 3 years before the minimum as the best indicator, our prediction for the next solar maximum is $R_{max} = 121 \pm 33$. A very similar result is obtained when solar cycle no. 19 is excluded: $R_{max} = 122 \pm 31$. If we repeat the procedure using data from solar cycles nos. 1-23 (up to the year 2008) to predict R_{max} of the solar cycle no. 24, we obtain $R_{max} = 134 \pm 33$. However, the actual value was 116.4 (Table 2). We emphasize that we should not directly compare our prediction of 121 with the value 116.4 and make a conclusion about the fact which solar cycle is or will be stronger. It is important to consider the RMSE values (note that the magnitude of the RMSE for both predicted R_{max} values are the same) which give the lower and upper limits for the predicted value. Our method predicts for solar cycle no. 24 the $R_{max} = 134 \pm 33$ and for solar cycle no. 25 the $R_{max} = 121 \pm 33$. Thus for the solar cycle no. 24 the predicted values are in the range 101–167 and for the solar cycle no. 25 in the range 88–154. Note that differences between predicted values of solar cycle no. 24 and solar cycle no. 25 are not a consequence of the fitting procedure (at least the effect is not significant). Namely, with 24 points (see Figure 3) we obtained Equation (7), and with 23 points (the cycle which we want to predict, no. 24, is omitted) the obtained expression is similar. The main reason for the difference are the values of R_{min-3}. In the prediction of the solar cycle no. 24 the value 36.0 for the epoch 12/2005 was used (3 years before the minimum of the cycle no. 24 which was at the epoch 12/2008), whereas in the prediction of solar cycle no. 25 the value 28.5 for the epoch 12/2016 was employed (3 years before the minimum of the cycle no. 25 which was at the epoch 12/2019), see also Table 1 . The used R_{min-3} input value for predicting the solar cycle no. 25 is lower and consequently also the calculated lower and upper limits, which indicate the possibility that the upcoming maximum will be lower. This is probably in accordance with the general middle-term lowering of the solar activity after the secular maximum in the 20th century and consistent with the Gleissberg period (the time scale of about a century) of the solar activity. However, the indication of lower upcoming minimum must be taken with some caution, as one should compare the trend of prediction for many cycles (not only cycle no. 24 and no. 25) with the real values to see if the method is capable to track the real trend, as was done in present work.

We now compare our prediction results with some early predictions found in the literature. So, Du (2020) applied the precursor method using the preceding minimum of *aa* geomagnetic index and forecasted the maximum of the solar cycle no. 25 to be $R_{max} = 151 \pm 17$, which is about 30% larger than the previous maximum, and also larger than our prediction, but still within 1σ error on both sides. On the other hand, Miao, Wang, Ren, & Li (2020) predicted $R_{max} = 122 \pm 33$ for the maximum of the cycle no. 25 using a combination of the Ohl's prediction method and minimum *aa* geomagnetic index. This forecasted value is very close to our predicted value (Equation 7).

It is worth to note that Petrovay (2020) gives the formulae for the minimum – maximum method based on solar cycles nos. 1–24, excluding cycle no. 19, for the cases R_{max} vs. R_{min} and R_{max} vs. R_{min-3}. The formulae are given in the review of Petrovay (2020) as Equations (10) and (11), which correspond to our Equations (4) and (5). We note, however, that the two sets of formulae are similar but not equal and that Petrovay (2020) does not provide the errors of the linear least-square fit parameters, which are calculated and used in present work. Based on the data available at the time of writing the review, Petrovay (2020) obtained that the maximal possible value for the next solar maximum is $R_{max} = 147$, which is now put to the lower values.

We can also raise the question why the predictor 3 years before the minimum gives the highest correlation coefficient CC, implying the most reliable epoch for the prediction, and to understand the importance of the value 3 years before the minimum which will determine the maximum of the next cycle. However, before discussing in detail the two papers (Cameron & Schüssler, 2007; Svalgaard et al., 2005) and the lines of reasoning of their authors, we briefly repeat some important ingredients of the self-exciting oscillating dynamo model of the Babcock-Leighton type. The model, along with later modifications and improvements, assumes that the differential rotation winds up the large-scale dipolar global poloidal field. This global poloidal magnetic field prevails the large-scale distribution of the surface field around solar activity minimum. The wound up poloidal field produces subsurface toroidal field which later moves across the surface and manifests itself as the sunspot activity of the next cycle. The strength of the polar magnetic field during the declining phase of one solar cycle is considered to be a sign of the highest amplitude of the solar activity in the next cycle. It is important to point out that this process takes place in the layers just below the solar surface. So, it can be directly observed during the time embracing the previous cycle. Consequently it is plausible to assume that the maximal value of the reversed polar magnetic field produced after the solar maximum will be a good precursor for the amplitude of the poloidal field from which the next

toroidal field will be produced by the differential rotation. So, the precursor method based on the polar field appears to be satisfactorily rooted in solar physics.

Svalgaard et al. (2005) made a solar maximum prediction based on the correlation between the amplitude of the solar magnetic dipole moment at the time 3 years before the minimum of activity with the amplitude of the next solar maximum. During the activity minimum the polar magnetic field attains maximal values and it changes polarity during the maximum of activity. The cause of this magnetic field reversal is the motion of unipolar magnetic flux from lower/medium latitudes towards the poles. This new flux cancels the flux of the opposite polarity already present there and the new magnetic field of the opposite polarity is eventually generated at the poles (Wang, Nash, & Sheeley, 1989). The new activity begins to destroy the polar magnetic field yielding the phenomenon of the strongest polar field during the time of approximately 3 years before the cycle minimum. Using the average magnetic field value during this time interval as the precursor for the next solar cycle, Svalgaard et al. (2005) succeeded to predict the maximal amplitude of the 24th solar cycle rather well, although the predicted value was lower by 10% compared to the really measured value.

Cameron & Schüssler (2007) investigated efficiency and reliability of solar cycle forecasting testing various methods based on precursors and magnetic flux transport models. As a predictor they used the magnetic flux protruding over the solar equator. This is justified by the fact that in the Babcock-Leighton dynamo model this quantity is related to the global dipole magnetic field from which the toroidal field for the next solar cycle is produced. As proxies for solar activity they used several measured and calculated quantities, such as sunspot number, sunspot area, polar field, equator flux and dipole component. These authors have concluded that the activity level 3 years before sunspot minimum is the best precursor for prediction of the amplitude of the subsequent maximum.

Cameron & Schüssler (2007) also offered an explanation about the origin of the predictive skill of the methods they used. The authors claim that to understand the predictive ability of the activity level in the descending phase of the activity cycle it is not needed to establish a physical relation between the surface magnetic manifestations in the previous and subsequent solar cycles. It is enough to embrace the two well-known properties of the sunspot number series. The first one is the concept of extended solar cycle (Harvey, 1992). This concept emphasizes the observational fact of the simultaneous appearance of sunspots at high latitudes, belonging to the new cycle, and at low latitudes, belonging to the previous, declining cycle. The second property is the Waldmeier effect (Brajša et al., 2009; Waldmeier, 1935) which relates the ascending time of a cycle toward its maximal phase and

the highest amplitude of the maximum. The Waldmeier effect means that stronger cycles rise faster towards their maxima. A combination of these two effects results in a systematic temporal shift of the minimum between the two subsequent cycles with various amplitudes, when using the activity indices which are averaged over latitudes (e.g., the sunspot number or the sunspot area). The final outcome of these two effects is the earlier occurrence of the minimum if the next cycle is stronger than the previous one, and the later occurrence of the minimum for the opposite case (a weaker following cycle). So, a higher level of activity 3 years before the minimum is a logical and necessary consequence of a statistically earlier epoch of the minimum. This line of reasoning can also help to understand yet another observational finding, namely the fact that the stronger cycles are statistically mostly preceded by the shorter cycles (Hathaway, Wilson, & Reichmann, 1994; Hathaway et al., 1999; Hathaway, Wilson, & Reichmann, 2002). So, it is not needed to search for a physical mechanism which would relate surface phenomena observed in subsequent solar cycles to justify the prediction using the precursors in the decreasing phase of activity. Cameron & Schüssler (2007) just offered a simple empirical explanation for the very useful "3 years before the minimum" predictor.

Finally, we note that the main problem in the reliability of the solar activity forecasting is the influence of the non-linear effects in the solar dynamo, which plays the major role in establishing and maintaining solar activity cycle (Hanslmeier et al., 2013). According to the present knowledge, solar activity cycle and the underlying solar MHD dynamo show properties on the edge of a chaotic process (Hanslmeier, 2020). So, the predictability is limited and the non-linear effects are the main source of uncertainty of any prediction.

5 | SUMMARY AND CONCLUSIONS

Our prediction for the maximum of the cycle no. 25 gives $R_{\max} = 121 \pm 33$. This result is based on the modified minimum - maximum method, correlating the level of activity 3 years before the minimum with the subsequent maximum. For this procedure the correlation coefficient is highest and inclusion or exclusion of the somewhat special cycle no. 19 does not influence the predictive skill. The reliability of the "3 years before the minimum" predictor is experimentally justified by the largest correlation coefficient and sufficiently explained with the two empirical well-known findings: the extended solar cycle and the Waldmeier effect.

So, we conclude that the next solar maximum will be of the similar amplitude as the previous one, or even something lower. This conclusion is based on the fact that the same method predicted a larger value for the maximum amplitude

of the 24th solar cycle compared to the prediction for the 25th solar cycle, and taking into account the general lowering of the solar activity consistent with the Gleissberg cycle.

This prediction is possible now when it is well established that the last solar minimum took place in December 2019, based on the smoothed monthly total (both solar hemispheres taken together) sunspot number.

Finally, we successfully tested the modified minimum-maximum method to reproduce some of the earlier maxima in order to check the accuracy of the predictions. The maxima of solar cycles 21-24 were calculated using the previously available data. The 3 years before the minimum method is preferred over the simple minimum–maximum method, since it has better correlation coefficients and lower statistical errors.

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APPENDIX A: ADDITIONAL MATERIAL

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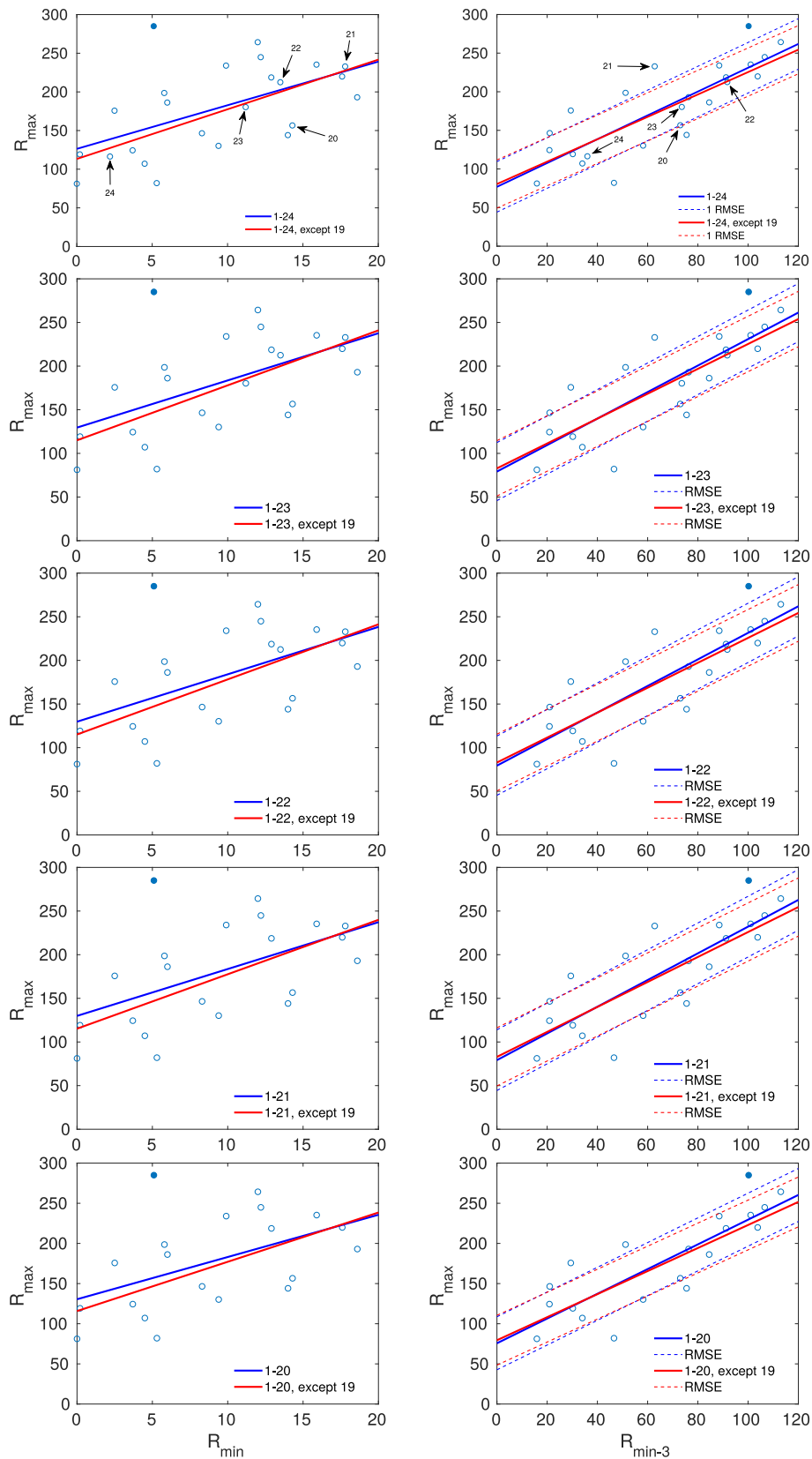Charbonneau, P. 2020, June, *Living Reviews in Solar Physics*, *17*(1), 4. doi:

FIGURE A1 The peak smoothed monthly sunspot number in solar cycle maxima as a function of the same quantity in the preceding solar minimum, for solar cycles nos. 1-24 (first row), 1-23 (second row), 1-22 (third row), 1-21 (fourth row) and 1-20 (last row), for min–max method (left column) and min-3–max method (right column). Least-square fits obtained with and without solar cycle no. 19 are presented with different colors, as indicated in the legend. The value for the solar cycle no. 19 is represented with the filled circle, while all other data points are represented with open circles.

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